Theoretical Analyses of Electrothermal Thrusters with Supersonic Heat Addition

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Theoretical performance analyses are presented for an electrothermal thruster using lithium propellant with a combination of radioisotope and electrical energy sources. This thruster concept allows the lithium propellant to condense in the nozzle and uses the latent heat of vaporization for increasing the exit velocity. Theoretical analyses were done for nucleation only, and for nucleation with subsequent particle growth. Significant increases in specific impulse are predicted with particle growth. For both cases, special nozzle contours are required, with small nozzle angles of the order of 1°. Specific impulses of greater than 500 sec are predicted with plenum temperatures below 2000°K and with theoretical electric power/thrust ratios of less than 3 w/mlb.

Introduction

THE performance of existing resistojets is limited to specific impulses below 300 sec with propellants of interest such as ammonia.[1] A higher specific impulse is required if resistojets are to be competitive with other propulsion systems for future applications. The analyses reported here show that resistojet specific impulse might be increased greatly by means of supersonic heat addition.

A fundamental limitation to the specific impulse of conventional resistojets is the practical limit for plenum temperature, which depends on the materials of construction and on the type of propellant. However, if more heat were added to the supersonic stream, it is theoretically possible to increase the exhaust velocity without incurring excessive materials temperature.

A number of means of supersonic heat addition were considered in the present study. Of these, the most attractive means appears to be the partial condensation of the vapor propellant in portions of the supersonic stream. For substantial amounts of heat addition by this means, the propellant must have a relatively high heat of vaporization. In addition, the propellant molecular weight must be low in order to obtain a high exhaust velocity. A brief review of propellants indicated that lithium has superior properties for this thruster concept. For example, if 50% of the lithium vapor were condensed in the supersonic stream, the total temperature would be increased by about 3000°K. With a plenum temperature of 2000°K and 50% condensate, the ideal specific impulse would be approximately 550 sec. Therefore, the present study was devoted exclusively to thruster designs with lithium propellant.

The Radioisotope/Resistojet Design Concept

Only a fraction of the total propellant flow can be expected to condense, therefore an appreciable amount of heat of vaporization would leave the thruster in the vapor exhaust. Early studies[2] of the condensing-flow electrothermal thruster concept showed that the power/thrust would be high if the heat of vaporization were supplied by electric heaters. If the heat of vaporization were supplied by a thermal heater such as a radioisotope capsule, then the electric power/thrust would be competitive with conventional resistojets. A schematic diagram of the radioisotope/resistojet concept is shown in Fig. 1.

The analyses reported here have been directed toward the radioisotope/resistojet concept, although much of the work is applicable to other thruster design concepts as well.

In addition to specifying lithium as the propellant, a nominal thruster mass flow rate of 0.0074 g/sec was used throughout the analyses. At a specific impulse of 600 sec, the thrust would be 10 mlb.

Basic Theory of Heat Addition to Supersonic Stream

The increase in velocity of a gas flowing through a nozzle follows the first law of thermodynamics:

$$h_0 = h + v^2/2 \tag{1}$$

where h_0 is the total enthalpy, h is the stream static enthalpy, and v is the velocity of the gas in the nozzle. With careful choice of nozzle geometry, heat may be added to the stream to increase the total enthalpy, thereby providing a velocity increase. Heating of the flow in the supersonic section of the nozzle was found to be more desirable than in the subsonic section since the over-all size of the nozzle would be smaller.

It is well known that if heat is added to a constant area of supersonic flow, the Mach number will decrease toward unity. However, if the nozzle area is increased at a sufficient rate, the additional energy will cause an increase in velocity, as given by[3]

$$\frac{dv}{v} = \left(\frac{1}{M^2 - 1}\right)\left[\frac{dA}{A} - \left(1 + \frac{\gamma - 1}{2} M^2\right)\frac{dT_0}{T_0} - (1 + \gamma M^2)\frac{d\dot{m}_v}{\dot{m}_v}\right] \tag{2}$$

where A is the cross-sectional area of the stream, γ is the specific heat ratio, M is the Mach number, T_0 is the total temperature, and $\dot{m}_v$ is the mass flow rate of the vapor stream. From inspection of Eq. (2), it is evident that the stream velocity in supersonic flow will increase if

$$dA/A > \{1 + [(\gamma - 1)/2]M^2\}(dT_0/T_0) \tag{3}$$

For condensing flow, $d\dot{m}_v$ is negative; therefore the stream

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velocity will increase if

$$\frac{dA}{A} > \left(1 + \frac{\gamma - 1}{2} M^2\right) \frac{dT_0}{T_0} - (1 + \gamma M^2) \frac{|d\dot{m}_v|}{\dot{m}} \quad (4)$$

The theoretical improvement in exhaust-velocity using heat addition in the nozzle is shown in Fig. 2, where T_{02}/T_{01} is the ratio of stagnation temperature in the nozzle to stagnation temperature in the plenum. It should be noted that although the total temperature in the nozzle can be several times greater than the total temperature in the plenum, the stream static enthalpy (hence static temperature) can be lower due to the increase in velocity [Eq. (1)].

Basic Theory of Nucleation

By proper choice of plenum temperature and pressure, condensate nuclei will begin to form at an appreciable rate in the supersonic section of the nozzle. The plenum temperature and pressure must be chosen such that the conditions in the throat of the nozzle will be below the critical temperature and pressure of the propellant. The propellant vapor was assumed to be a perfect ideal gas with isentropic expansion to the point where nucleation begins, as shown in Fig. 3.

For lithium vapor, an appreciable nucleation rate was found to begin at a saturation pressure ratio of about four. The critical-droplet radius r^* for homogeneous nucleation is given by[4]

$$r^* = 2\alpha/\rho_l RT \ln(p_v/p_s) \quad (5)$$

where α is the surface tension, ρ_l is the liquid density, R is the gas constant, T is the vapor temperature, p_v is the stream static pressure, and p_s is the saturation pressure.

The critical-sized droplet can be thought of as that having the maximum of Gibbs free energy. Thus, if a particle is less than critical size r^*, it will re-evaporate; on the other hand, if it is greater than r^*, it will have the tendency to continue growing.

The rate of nucleation J_x of critical-sized droplets in a unit volume of vapor is[4]

$$J_x = \left(\frac{p_v}{kT}\right)^2 \frac{1}{\rho_l} \left(\frac{2\mu\sigma}{\pi N_A}\right)^{1/2} \exp\left(-\frac{4\pi\sigma r^{*2}}{3kT}\right) \quad (6)$$

where k is Boltzmann's constant, N_A is Avogadro's number, and μ is the molecular weight. The number of nuclei S_J formed per unit distance travelled in the nozzle is

$$S_J = J_x \Delta V \Delta t \quad (7)$$

where ΔV is the volume of an interval in the nozzle and Δt is the time required for the flow to traverse the interval Δl:

$$\Delta V = \bar{A}\Delta l = \bar{A}\bar{v}\Delta t \quad (8)$$

where $\bar{A}$ is the average cross-sectional area of the nozzle in the interval, and $\bar{v}$ is the average velocity in the interval.

Growth of Condensation Nuclei

Once the critical-sized droplets are formed, they begin to grow as a result of collisions with the vapor molecules, some

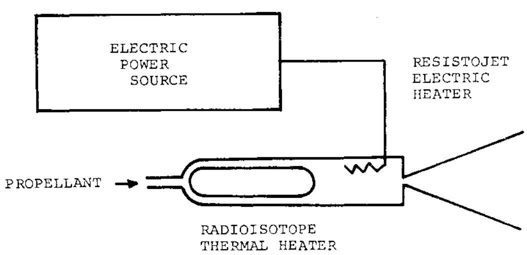

Fig. 1 The radioisotope/resistojet concept.

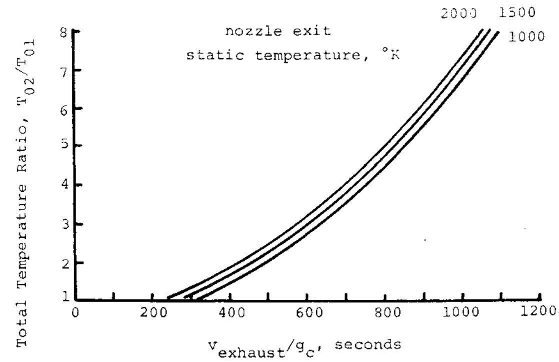

Fig. 2 Theoretical exhaust velocity in suerpsonic flow of lithium with supersonic heat addition. Plenum total temperature, T_{01} = 2500°K; stream total temperature, T_{02}.

of which condense on the surface of the droplets. Only a small fraction α of the atoms will stick to the nuclei; therefore the mass rate of arrival Γ_v must be multiplied by a sticking coefficient α to obtain the rate of mass increase Γ_a per unit area:

$$\Gamma_a = \alpha\Gamma_v \quad (9)$$

where the mass rate of arrival for a Maxwellian velocity distribution[5] is

$$\Gamma_v = p_v/(2\gamma RT)^{1/2} \quad (10)$$

The sticking coefficient α is[5,6]

$$\alpha = \frac{c_{pl}(T_p - \bar{T})}{h_{fg}[1 - (2\sigma/r_p\rho_l h_{fg})]} \quad (11)$$

where c_{pl} is the specific heat of the liquid, T_p is droplet temperature, $\bar{T}$ is the average vapor temperature in the nozzle increment, h_{fg} is the heat of vaporization, α is the surface tension, r is the droplet radius, and ρ_l is the liquid density.

It should be noted here that the sticking coefficient α accounts for both the reflected incident atoms, and for atoms that are evaporated from the droplet. The expression for α given by Eq. (11) is valid only for quasi-steady growth models.

The rate $\dot{m}_c$ at which nucleation and growth occurs in any particular nozzle interval n is

$$\dot{m}_c = \dot{m}_J + \dot{m}_{a0} + \dot{m}_a \quad (12)$$

where $\dot{m}_J$ is the mass condensed due to fresh nucleation in the nozzle interval n, $\dot{m}_{a0}$ is the mass condensed in the nozzle

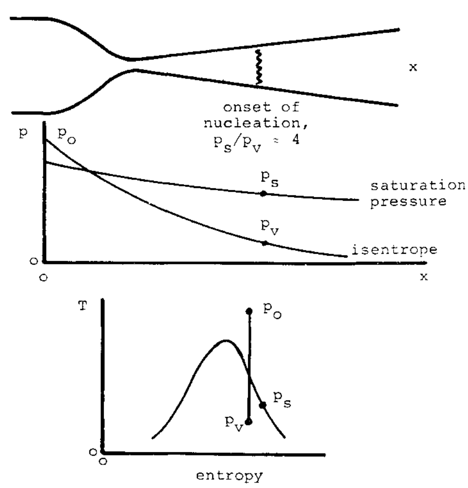

Fig. 3 Onset of nucleation in condensing flow.

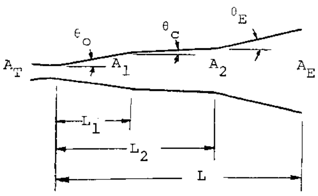

Fig. 4 Nozzle design parameters.

interval n onto droplets which were originally formed by nucleation in the first nozzle interval, and $\dot{m}_a$ is the mass condensed in nozzle interval n onto droplets which were originally formed by nucleation in the nozzle intervals 2 to $n - 1$.

Mass $\dot{m}_J$ due to fresh nucleation is

$$\dot{m}_J = \rho_l(\tfrac{4}{3}\gamma r^{*3})\Delta V J_x \tag{13}$$

where r^* is given by Eq. (5), ΔV is the volume of the nozzle interval n, and J_x is given by Eq. (6).

Mass $\dot{m}_{a0}$ condensed onto droplets that originated in the first nozzle interval is

$$\dot{m}_{a0} = \Gamma_a S_{J1}\theta \tag{14}$$

where Γ_a is given by Eq. (11), S_{J1} is given by Eq. (7) evaluated in the first nozzle interval, and θ is the collision cross section:

$$\theta = 4\pi(r_m + r)^2 \tag{15}$$

where r_m is the lithium atom radius, and r is the drop radius accounting for the growth through nozzle intervals 2 to $n - 1$.

Mass $\dot{m}_a$ condensed onto droplets that originated in nozzle intervals 2 to $n - 1$ is

$$\dot{m}_a = \Gamma_a \left(\sum_{i=2}^{n-1} S_{Ji}\right) \bar{\theta} \tag{16}$$

where $\bar{\theta}$ is an average cross section for droplets in the nozzle interval n, which is computed using $\bar{r}$, an average of droplet radii for the drops originating in nozzle intervals 2 to $n - 1$, in Eq. (15). By approximation, $\bar{r}$ was determined by assuming that this class of drops had grown to $\frac{1}{2}$ the mass of drops that originated in the first interval:

$$\bar{r} = (\tfrac{1}{2})^{1/3} r \tag{17}$$

When the vapor condenses, the latent heat of vaporization is released to the vapor stream. This heat causes the total temperature to rise, as given by

$$T_0 = (\dot{m}_c h_{fg})/(\dot{m}_{\text{tot}}\bar{c}_p) \tag{18}$$

where $\dot{m}_c$ is given by Eq. (12), $\dot{m}_{\text{tot}}$ is the total propellant flow rate, and $\bar{c}_p$ is an average specific heat for liquid and vapor:

$$\bar{c}_p = f c_{pl} + (1 - f)c_{pv} \tag{19}$$

where f is the fraction of total propellant flow which has been condensed. In Eqs. (18) and (19), it is assumed that the temperature rise of the droplet is the same as the increase in total temperature of the vapor stream. This approximation was considered adequate for the purposes of this preliminary analysis, but future work should include a detailed heat balance of the droplet growth process for determination of the actual rise in droplet temperature. According to a previous theoretical analysis,[6] the drop temperature should be close to the saturation temperature for the local stream static pressure. Since stream static pressure is continuously decreasing through the nozzle, the droplet temperature may actually decrease, and if this is true then the assumption used in Eqs. (18) and (19) is conservative. In addition, future work should include the effect of momentum interchange between the droplets and the vapor stream on the stream total-temperature increase.

The combination of the total temperature change, the condensation fraction, and the geometry (area change of the nozzle) can be used in Eq. (2) to determine the velocity in an increment of the nozzle.

Method of Analysis

Analysis of the condensing nozzle flow was done on a CDC-6400 digital computer. An iterative method was used to solve the theoretical equations over many very small intervals of nozzle length:

1) A conical nozzle geometry was assumed, with switching controls to change the expansion angles at stations such as the onset of appreciable nucleation (see Fig. 4).

2) Plenum temperature and pressure were assumed.

3) Isentropic flow was calculated along the nozzle to the first interval where appreciable nucleation occurred.

4) The temperature T_1, the pressure p_1, and the Mach number M_1 were assumed constant through the first interval during the first iteration.

5) For the first interval, the condensate mass, the total-temperature rise, and the velocity change were calculated based on the given nozzle area change.

6) From the velocity change, T_2, p_2, and M_2 were determined for the end of the first interval.

7) Returning to the start of the first interval, the condensate mass was recalculated using the average values $(T_1 + T_2)/2$ and $(p_1 + p_2)/2$.

8) The iterative process was repeated until the average properties of the interval $(T_1 + T_2)/2$ and $(M_1 + M_2)/2$ were negligibly different from those of the previous iteration.

The following assumptions were made in the analysis:

a) The lithium vapor was assumed to be a perfect gas, which is very nearly correct since lithium is monatomic.

b) The drops were assumed to travel at the vapor stream velocity. Experimental measurements have shown that particles of less than 10^{-5} cm diam move at stream velocity,[5] and that particles as large as 10^{-4} cm diam move at approximately 99% of the stream velocity.[6] Droplet diameters calculated in the present analysis were less than 10^{-6} cm; therefore, the stream velocity assumption appears to be valid.

c) Boundary-layer thickness and friction were assumed negligible. Because of the nozzle dimensions, the favorable pressure gradient, and the accelerating flow, this assumption may be valid, but should be verified in future work.

d) Convective heat transfer from the stream to the nozzle walls was not considered. A regenerative propellant flow design might be used to prevent excessive nozzle wall temperature. Future work should include this heat transfer in the nozzle-flow heat balance.

e) Radiation heat transfer from the walls to the droplets in the stream was not considered. With high condensate fractions, this might return appreciable heat to the nozzle flow.

Nozzle Design and Performance Assuming No Particle Growth

In the early phases of the present study, a conical nozzle having a constant expansion angle was considered. It was soon found that the condensation fraction increased as the angle of the nozzle was decreased. In fact, it was found that an extremely small nozzle half-angle of about 0.5° gave the highest fraction of condensate. For example, a nozzle half-angle of 0.5° gave approximately twice the amount of condensation as a nozzle having a 5° expansion. However, nozzle designs with these small expansion angles required extremely long nozzle lengths to convert the thermal energy of the lithium propellant into kinetic energy. It was also found that since most of the condensation occurred in a relatively small section of the nozzle, the nozzle expansion could be increased after the high-condensation region. This increased angle after the condensation region greatly shortened the nozzle without much loss in performance. Furthermore, it was found that the nozzle could be shortened further if the angle θ_0 upstream of the condensation was increased (to between 1° and 3°), since it had no effect on the condensation. A typical nozzle design with these features is shown in Fig. 4. The stations where these nozzle angle changes took place was found to have a strong effect on the total length of the nozzle. If the larger angle θ_0 was allowed to penetrate the condensation region, the nucleation rate would increase faster than with the smaller angle θ_c in the condensation region. Thus, the nozzle would be further shortened if θ_0 was continued to where the nucleation rate approached the maximum value. However, if this θ_0 was continued too far, the region of condensation in the nozzle would be reduced in extent, thereby reducing the condensation fraction.

The effects of nozzle angles on the condensation fraction, the nozzle length, and the specific impulse for a particular set of plenum conditions is shown in Table 1. In each case, the nozzle-angle change from θ_0 to θ_c was done when the nucleation rate reached the values shown in the table. The nozzle angle was changed from θ_c to θ_E when the critical radius of the nuclei decreased to 1 Å. The nozzle length was terminated when the static temperature approached 60°K.

From inspection of Table 1, it is evident that nozzle geometry has a marked effect on nozzle length. It is notable that the results reported here do not necessarily represent optimum nozzle geometries. A fully optimized nozzle geometry would be contoured rather than have the fixed cone angles shown in Table 1.

A summary of nozzle performances is shown in the upper half of Table 2 for ranges of plenum temperatures and pressures. The vaporizer temperature is shown for each plenum pressure; that is, the vaporizer is assumed to generate the plenum pressure. Although these results do not represent fully optimized nozzle geometries, some general trends of nozzle performance can be seen. Nozzle length is greatly reduced when higher plenum pressures (corresponding to higher vaporizer temperatures) are assumed. As expected, specific impulse is increased by higher plenum temperature.

Electric power/thrust ratios, based on the radioisotope/resistojet concept shown in Fig. 1, are also listed in Table 2. In this concept electric power is used only to heat the propellant vapor from the vaporizer temperature to the plenum temperature. At a specific impulse of 500 sec, an all-electric resistojet with an efficiency near 100% would have a power/thrust ratio of 11 w/mlb; therefore, the theoretical performance of the radioisotope/resistojet concept is most attractive even with no particle growth.

Nozzle Design and Performance with Particle Growth Included

Experiments[5–8] done on condensation in nozzles have shown that the droplets grow after nucleation. The growth of the droplets was found to considerably increase the total fraction of the condensate in designs of nozzles similar to those shown in the nongrowth case. It was found, however, that the geometry for the nongrowth case would not be the best design for the nozzle with growth.

Table 1 Effect of nozzle geometry on nozzle length, with no particle growth

θ_0, deg	Ja, cm³-sec	L_1, cm	θ_c, deg	L_2, cm	θ_E, deg	cond. fract., F	L, cm	I, sec
1.0	10^{18}	0.30	0.34	4.49	5.73	0.271	10.20	485
2.0	10^{22}	1.04	0.57	1.55	0.57	0.245	9.62	477
1.0	10^{21}	3.17	0.34	4.29	5.73	0.269	5.87	484
1.0	10^{21}	0.46	0.57	3.07	11.46	0.260	4.28	481
2.0	10^{22}	1.05	0.34	1.54	5.73	0.235	2.98	473
2.0	10^{22}	1.04	0.86	1.54	14.30	0.224	2.19	470

Table 2 Summary of nozzle performance plenum temperature, T_p; plenum pressure, P_p; vaporizer temperature, T_v; total temperature, T_0; fraction condensed, f; exit/throat area ratio, A_E/A_T; nozzle length, L; specific impulse, and electric-power/thrust ratio, P/F

T_p, °K	P_p, atm	T_v, °K	T_0, °K	f	A_E/A_T	L, cm	I, sec	θ, deg	P/F, w/mlb
				Nucleation only					
1600	0.069	1300	3141	0.260	41.7	78.1	437	...	0.9
1800	0.069	1300	3277	0.247	49.2	43.0	447	...	1.5
1800	0.45	1500	3537	0.303	14.5	4.2	468	...	0.9
2000	0.069	1300	3252	0.202	81.4	32.2	454	...	2.1
2000	0.45	1500	3686	0.291	14.1	5.9	475	...	1.4
2000	1.0	1600	3784	0.313	5.4	4.9	480	...	1.1
2250	0.069	1300	3467	0.196	74.5	28.0	460	...	2.8
2250	0.45	1500	3835	0.269	14.9	5.9	485	...	2.1
2250	1.0	1600	3943	0.29	8.2	3.5	491	...	1.8
2250	2.0	1670	4090	0.308	5.4	2.9	477	...	1.6
2500	0.069	1300	3658	0.185	52.8	43.3	472	...	3.5
2500	0.45	1500	3982	0.249	15.7	5.7	494	...	2.7
2500	2.0	1670	4110	0.274	7.5	1.7	502	...	2.2
2500	5.0	1860	4265	0.31	2.3	1.3	510	...	1.7
				Nucleation and growth					
1600	0.069	1300	3472	0.334	3.49	4.0	462	1.0	0.88
1800	0.069	1300	3578	0.312	3.77	4.46	469	1.0	1.45
1800	0.45	1500	3741	0.351	2.21	0.92	480	1.0	0.85
1800	0.45	1500	3688	0.338	2.61	0.60	477	2.0	0.86
2000	0.069	1300	3732	0.302	3.59	4.37	479	1.0	1.99
2000	0.069	1300	3758	0.308	3.47	2.11	481	2.0	1.98
2000	0.45	1500	3868	0.333	2.33	1.02	489	1.0	1.39
2000	1.0	1600	3865	0.333	2.26	0.66	488	1.0	1.52
2250	0.069	1300	3960	0.300	2.73	3.28	498	1.0	2.60
2250	0.45	1500	3992	0.304	2.55	1.19	496	1.0	2.06
2250	1.0	1600	4080	0.326	1.86	0.51	499	1.0	1.77
2250	1.0	1600	4013	0.309	2.40	0.38	498	2.0	1.78
2500	0.069	1300	4028	0.259	3.56	4.57	499	1.0	3.38
2500	0.45	1500	4152	0.284	2.62	1.26	506	1.0	2.69
2500	1.0	1600	4243	0.3058	1.87	0.52	508	1.0	2.41
2500	2.0	1670	4278	0.313	1.92	0.39	514	1.0	2.20

After several attempts to modify slightly the geometry of the nongrowth design, it was found that the optimum design of the nozzle with growth would be quite different from its nongrowth counterpart. The required design changes in the nozzle are due to the following factors:

1) The over-all rate of condensation is increased. In order to utilize the increased heating of the stream, the nozzle angles in the condensation region must be increased beyond those of the nongrowth case.

2) Large nozzle exit angles cannot be used, because the rate of growth increases greatly when the stream temperature is low.

3) Nucleation and particle growth occur in different regions of the nozzle. The nucleation occurs as soon as the saturation pressure ratio is greater than 4, whereas growth occurs after nucleation. The heat released to the stream during growth tends to stop the nucleation. Further analyses may provide nozzle designs in which optimum conditions exist for both nucleation and growth.

As an initial step in developing the geometry for the nozzle, the performance was studied with a constant-angle expansion ($\theta = \theta_0 = \theta_c = \theta_E$), for a range of plenum temperatures and pressures. A summary of the results is shown in the lower half of Table 2. The specific impulse was found to be greater by about 5%, while the required length of the nozzle was found to be considerably shorter than for the nongrowth counterpart. It is noted that the effect of growth on improvement of performance was the greatest for the lower plenum temperature cases.

The best performance was found with $\theta \simeq 1°$. The best choice of θ was a function of the plenum pressure. If the plenum pressure was low (below $\frac{1}{2}$ atm), a slightly larger θ

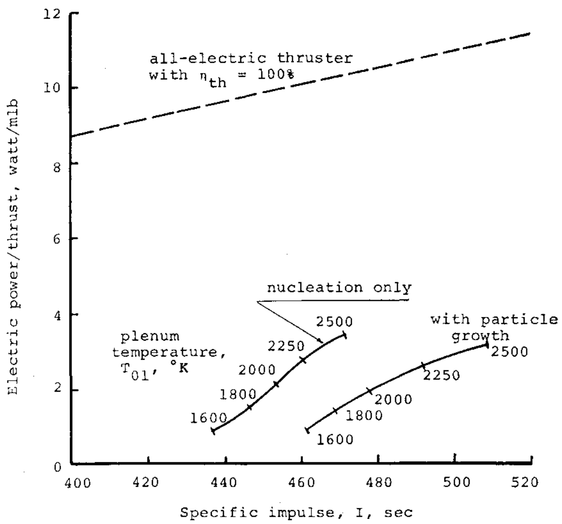

Fig. 5 Comparison of theoretical performance of lithium-condensing nozzle electrothermal thrusters with vaporizer temperature of 1300°K.

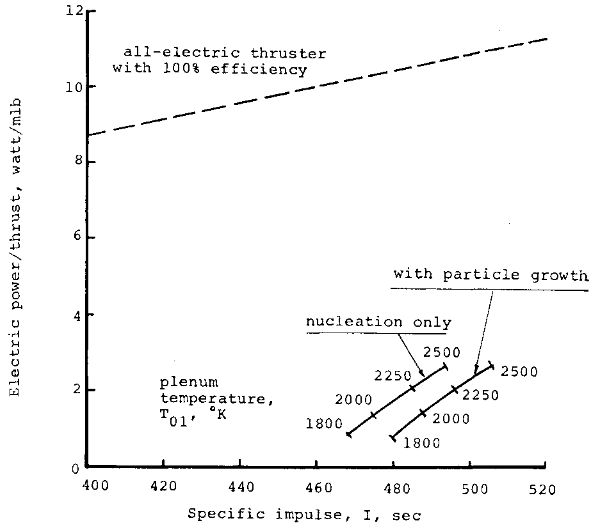

Fig. 6 Comparison of theoretical performance of lithium-condensing nozzle electrothermal thrusters with vaporizer temperature of 1500°K.

would increase the performance; while for higher plenum pressure, a slight decrease in θ improved the performance.

It should be noted that the work on the nozzle with growth is in the preliminary stages. Based on past experience on the geometry study of the nongrowth case, the specific impulse can be expected to increase by about 5 to 10% above those presented in Table 2 as the effects of geometry on the design are better understood. Thus, it is estimated that, with a plenum temperature as low as 2000°K, a theoretical specific impulse of 550 sec will be possible.

Summary of Results

Theoretical performance of the radioisotope/resistojet thruster concept with a lithium-condensing nozzle is shown in Figs. 5 and 6 for vaporizer temperatures of 1300 and 1500°K, respectively. Electric power/thrust ratios are shown for both the nucleation-only analysis, and for single-angle nozzles with particle growth. Although these results are preliminary, it is clear that particle growth provides markedly higher performance.

Conclusions

The theoretical analyses reported here indicate that the lithium-condensing radioisotope/resistojet thruster concept should have performance much superior to existing electrothermal thruster concepts. Specific impulse values of at least 500 sec appear possible with ideal electric power/thrust values below 3 w/mlb.

Particle growth provides definite improvement in theoretical performance. Further work is being done to develop nozzle geometries that optimize the competing processes of nucleation and ensuing particle growth. It is anticipated that theoretical specific impulse values as high as 550 sec will be found for fully optimized nozzle geometries with realizable vaporizer and plenum temperatures.

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